Geometry and Algebra in Ancient Civilizations. By B. L. van der Waerden. Berlin/Heidelberg/New York/Tokyo (Springer-Verlag). 1983. xii + 223 pp. 98 Figures. Index.

Reviewed by Frank J. Swetz

The Pennsylvania State University, Middletown, Pennsylvania 17057

In this book Professor van der Waerden has undertaken an ambitious task, namely to survey and comment upon the geometric and algebraic traditions of the ancient world. Building upon both archaeological and historical evidence, he has attempted to trace the evolution of two mathematical disciplines over a period of four thousand years (ca. 3000 B.C.–A.D. 1000). The scope of this examination spans the accomplishments of Neolithic Europe, Babylonia, Greece, Egypt, and the Hindu and Chinese Empires. Despite such a broad undertaking, the author, through his descriptive narrative, admirably accomplishes his purpose of surveying the content and methods of early algebra and geometry. But there is much more to this book than a mere compilation of facts. This study serves as a vehicle to propound some new theories (and resurrect some old ones) that are startling, thought-provoking and, perhaps, a bit disturbing in their implications. Van der Waerden hypothesizes that the mathematical sciences had their origins in Neolithic Europe with a mysterious but highly capable "Beaker people" whose involvement with the "Pythagorean proposition" and "Pythagorean triples" established ritual origins for all subsequent mathematics.

Attempting to trace mathematical developments in prehistory is a difficult undertaking and rests largely on speculation, but some research exists to help shape such conjecture. Archaeological and astronomical research by a score of investigators has established the existence of Neolithic lunar and solar observatories scattered throughout Northwestern Europe. Physical surveys of many of these observatory sites by Oxford's Alexander Thom has led him to believe that their builders possessed a geometry that incorporated the use of the right triangle and right-triangle theory i.e., Pythagorean triples. Further, the construction of many such Neolithic observatories has been identified with a race of early Europeans known as the "Beaker people," so identified for their special forms of pottery. Little is known of these people except that they probably immigrated to the British Isles about 2500 B.C., buried their dead in mounds, and had distinctive round heads. Using the findings of Thom and influenced by the work of A. Seidenberg on the ritual origins of mathematics, van der Waerden attempts to construct a theory of mathematical transmission and adoption from early Europe to China. This theory is established on the debatable premise that mathematical discoveries originate in one location and are transmitted elsewhere (p. 10). Throughout his book, van der Waerden pursues his protogenesis theory of mathematics by the use of intersocietal comparisons of mathematical ideas and techniques. That the

Beaker people apparently utilized Pythagorean triples in the construction of their observatories (2000 B.C.) and that certain sets of Pythagorean triples also appear at early dates in Egyptian, Greek, Babylonian, Hindu, and Chinese societies are construed as evidence of a common transmission (Chap. 1). Similar comparisons are made between Babylonian and Chinese algebraic processes and their use of geometric–algebraic solution formats (Chaps. 2 and 3); consideration of the solution of indeterminate equations in Greece, and India, and China (Chaps. 4 and 5); and the textual presentation of popular mathematics in Greece, Babylonia, Egypt, and China (Chap. 6). Greek influences are sought out in the work of both the Chinese mathematician Liu Hui (ca. 260) and the Indian astronomer Aryabhata (d. 746) (Chap. 7).

The theory of a common origin for early mathematics is certainly intriguing; however, the case made for it is weak and rests on selective scholarship. Van der Waerden apparently views mathematics as a purely intellectual activity, with its results springing from isolated human genius. In this work, he completely ignores the role of society in shaping mathematical thought and applications. Ancient Egypt, Babylonia, and China are examples of "hydraulic" societies, that is, societies highly dependent on agriculture and supporting systems of irrigation and astronomy within which mathematics was developed. Similar needs and conditions in these three societies resulted in similar empirically derived mathematics. While societal similarities are listed in the textual discussion (p. 45), their influence on mathematical thinking is not pursued. It is true that Pythagorean triples appeared and were used in different early societies, but they were apparently arrived at by different means: empirically in Neolithic Europe and India; by computational formula in Babylonia; and directly from the Pythagorean proposition in China. In several early societies, we know that "cord-stretchers" performed geometrical constructions in the layout of structures, a fact attesting to the rudimentary nature of surveying rather that the existence of a common mentor. Greece and China are cited as employing counting boards for numerical computation but the form and operation of these boards are quite different, the Greek version being a slab abacus employing counters, whereas the Chinese board is operated by means of a set of rods in such a manner as to form algorithmic procedures. In pursuing the argument for common origins, van der Waerden notes the similarities in the forms of ancient Egyptian, Babylonian, and Chinese mathematics texts—they are collections of specific problems accompanied by solution schemes. What better way to show an official how to solve a standard problem? Certainly, this format is not unusual. It has been employed in school lessons for thousands of years. In his attempts to establish a line of mathematical transmission eastward, the author frequently ignores anomalies that would seem to contradict this transmission. For example, the origins of a decimal numeration system are associated with Europe and noted as also existing in China—evidence of transmission; however, China's mathematical traditions are traced through Babylonia and the Babylonians used a sexagesimal system!

Van der Waerden's book is unique in that it extends the usual scope of historical mathematical examination to include European Neolithic accomplishments and early Chinese work in the same context as Greek–Babylonian traditions. However, his survey of Chinese mathematics and the conclusions he draws are limited mainly to selected contents of the *Jiu zhang suanshu* [*Nine chapters on the mathematical art*] (200 B.C.–A.D. 200) and the writings of Liu Hui. It is unfortunate that upon such limited exposure to the scope and the depth of early Chinese mathematics, doubts are raised in the author's mind on both its indigenous character and its origins. After each examination of the content and methods of Chinese mathematics, van der Waerden posits foreign influences (pp. 44, 60, 66). Similar biases were expressed by G. Loria earlier in this century. The concern of early Chinese mathematicians with right-triangle theory is certainly misunderstood as indicated by the statement. "Pythagorean triangles did not form an essential part of either Greek or Chinese mathematics: they were just playful additions, which enabled Diophantos and the author of the Nine Chapters to propose nice little problems about right-angled triangles" (p. 38). A use of right triangles and right-triangle theory lay at the heart of early astronomy and surveying and these two sciences were essential for the functioning of the Chinese Empire. In the ninth chapter, *kou ku,* of the *Jiu zhang suanshu,* sixteen problems consider various mathematical situations in which the Pythagorean theorem is applied. These problems are neither "playful" nor trivial in their conception or intent. In fact, this collection of problems remains the most complete and extensive work on right triangles from the ancient world. Liu Hui in his third-century revision of the *Jiu zhang* extended this collection to include nine additional problems involving complex surveying situations. Eventually, on the basis of their usefulness and importance, these nine problems were compiled into a separate work, *Haido suanjing* [*Sea island mathematical manual*], which remains today a classical work on right-triangle theory. Certainly a more profound examination and appreciation of China's mathematical accomplishments and early social milieu is warranted before conclusions about foreign influences can be reached; considering the physical isolation and xenophobic character of early Chinese civilization, the probability of foreign mathematical influences is certainly diminished.

The fact that van der Waerden's major premise is controversial should not detract from the merits of this work. After all, a little controversy, even in the history of mathematics, is valuable at times as it forces people to think about the issues in question, in this case, the origins of mathematics. This book, is informative and a helpful resource in attempting to understand the nature of early algebra and geometry. Many interesting problems from antiquity are presented and examined. Both a comprehensive bibliography (reference citations are given directly in the text) and a concluding chapter summarizing the theory of mathematical transmission would be welcomed additions to the book. Although this work fails to prove the existence of a "Garden of Eden" for mathematics, it nevertheless can be strongly recommended for library acquisition and personal reading.